University of Delaware
Discrete Mathematics Seminar

Double Covers

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Abstract

The usual notion of a double cover (from topology), specialized to the case of graphs, reads as follows: Let Γ and Γ' be ordinary graphs (undirected, without loops or multiple edges) and let $f : \Gamma \to \Gamma'$ be a homomorphism (so f maps vertices of Γ to vertices of Γ', while preserving adjacency). Then f is a *double cover*, i.e. a *cover of index 2*, if f is 2-to-1 both on vertices and on edges.

First example: The graph of a cube (8 vertices, 12 edges) doubly covers the complete graph K_4 (4 vertices, 6 edges). Second example: The graph of a dodecahedron (20 vertices, 30 edges) doubly covers the Petersen graph (10 vertices, 15 edges). Covers of index 2 enjoy properties not generally shared by covers of higher index.

I will describe some natural ways that graphical double covers arise in combinatorics and finite geometry, in particular leading to the construction of new finite projective planes.