Math 117

Exam 2

Spring 2008

Please write your answers in the space provided. Make sure your work is kept organized and neat. Read instructions for each problem carefully. This exam is worth 175 points. Please make sure your exam has 18 problems.

You must use the techniques presented in this class to answer the questions. All fraction, rational expressions and radicals must be reduced and simplified. Approximate only when directed. Only scientific calculators are allowed, no graphing calculators.

Name: _____

Section: _____

Section 10 meets MWF at 11:15
Section 11 meets MWF at 2:30

Instructor: B. Daley

Please show work for partial credit. Each problem is worth 10 points unless otherwise indicated.

1. (4 pts) Solve $x^2 + 9 = 0$ over the complex numbers.

_____

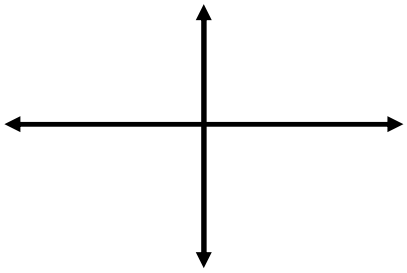
2. (4 pts) Determine the horizontal and vertical asymptotes for the rational function:

$$f(x) = \frac{2x+3}{x-6}$$

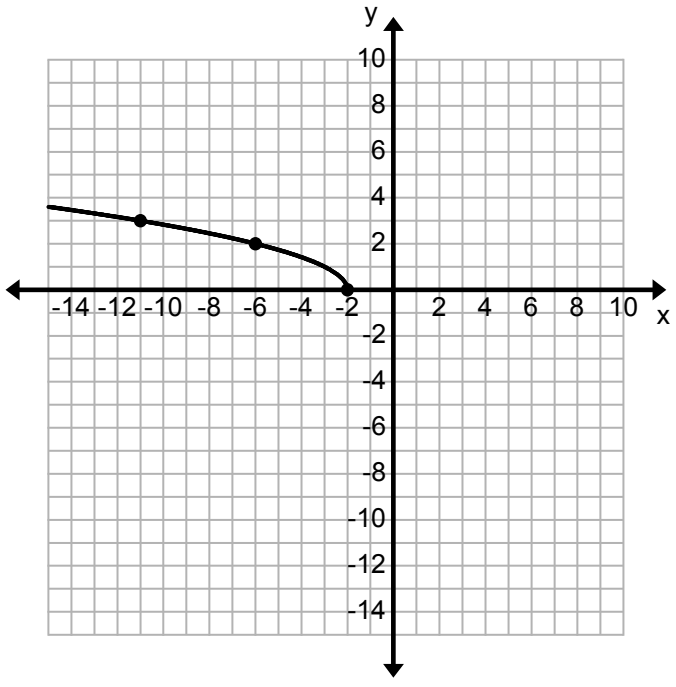
Horizontal: _____

Vertical: _____

3. (8 pts) Let $f(x) = ax^2 + bx + c$. Draw a sketch of $f(x)$ given the conditions $a > 0$ and $b^2 - 4ac < 0$.



4. (8 pts) The graph of f(x) is given below. On this graph, sketch $f^{-1}(x)$



5. (8 pts) Using interval notation, determine the domain of $f(x)=\sqrt{4-x^2}$

_____

6. Let $f(x)=x^2-3$ and $g(x)=\sqrt{x+1}$. Determine $(f\circ g)(x)$. Then determine the domain of $(f\circ g)(x)$.

$(f\circ g)(x)=$ _____

Domain: _____

7. Let $f(x)=\sqrt{x+1}$. Determine $f^{-1}(x)$ and its domain.

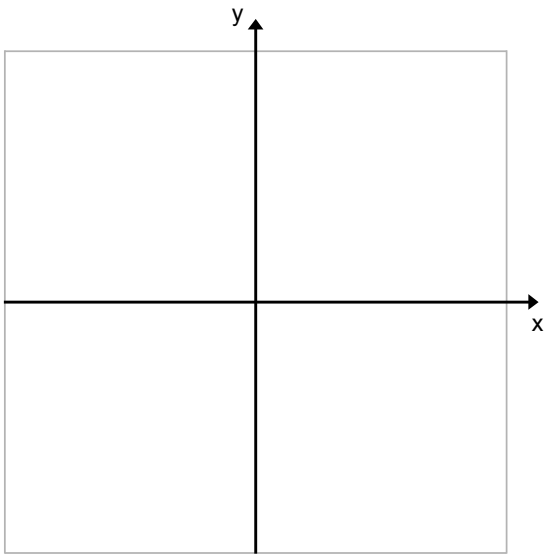
$f^{-1}(x)$: _____

Domain: _____

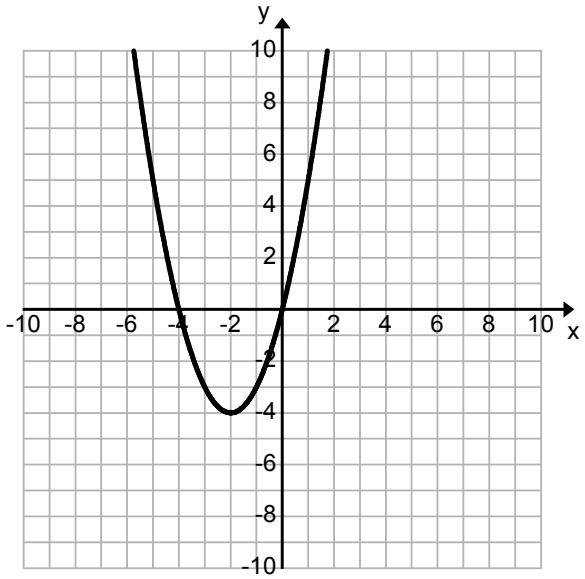
8. Given that $\boldsymbol{p(x)=x^3-3x^2-10x-6}$ and $\boldsymbol{p(-1)=0}$, determine the remaining zeros of $\boldsymbol{p(x)}$.

Zeros are: _____

9. Graph the polynomial function $p(x)=-x^3(x-2)^2(x+3)^2$ clearly indicating all x-intercepts, the behavior of the graph near these intercepts and the end behavior of the graph.

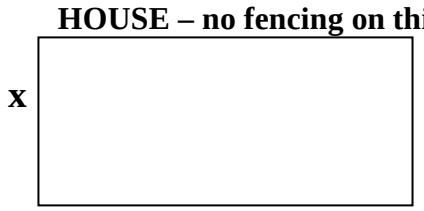


10. (8 pts) Sketch the graph of $g(x) = -f(x-3)$ given the graph of $f(x)$ below.



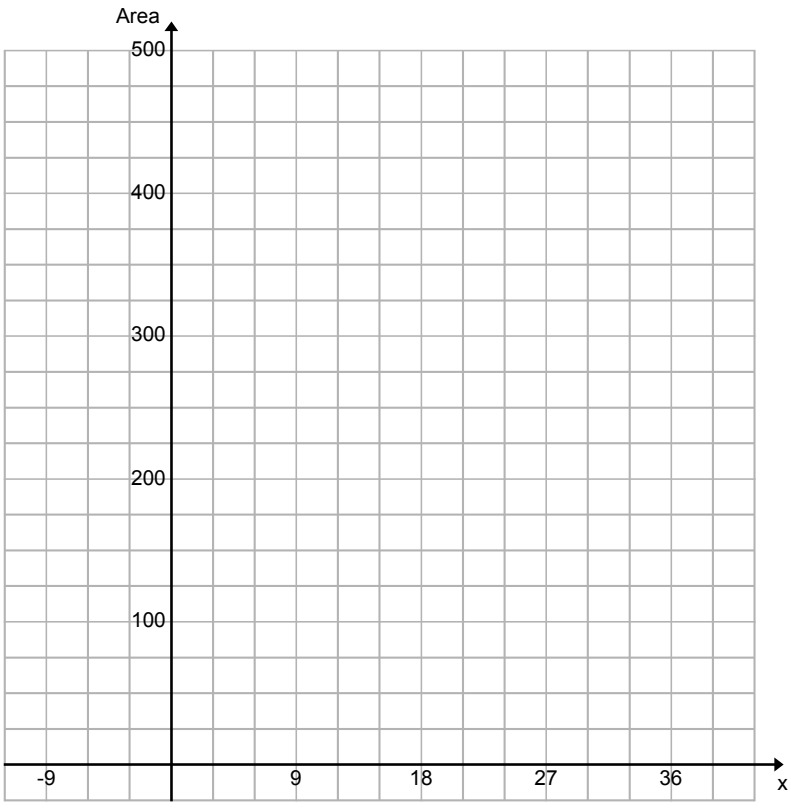
11. If $f(x) = \dfrac{3x+1}{2x-4}$ and $g(x) = \dfrac{1}{x}$, determine AND simplify $(f \circ g)(x)$. Your answer should be a simplified reduced rational expression.

12. (20 pts) Sarah wants to fence in a rectangular dog pen against her house. Since the house is to serve as one of the sides, she needs to fence only three sides. Let x be the length of one of the sides of the pen as indicated in the figure below:



A. If she uses 60 feet of fencing, determine the area in terms of x.

B. You should have a quadratic function. Find the x-intercepts and vertex and graph your area function below:



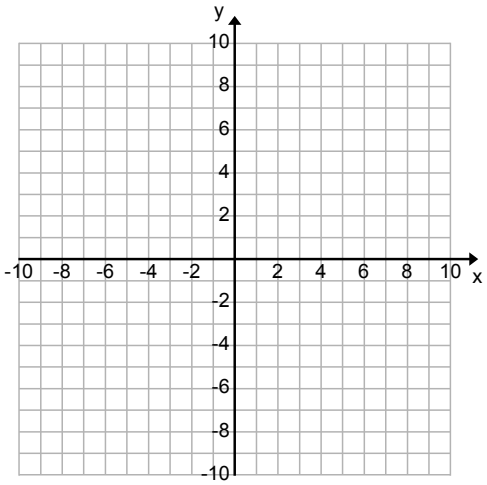
Vertex: _____

x-intercepts: _____

C. What is the maximum area that can be enclosed? _____

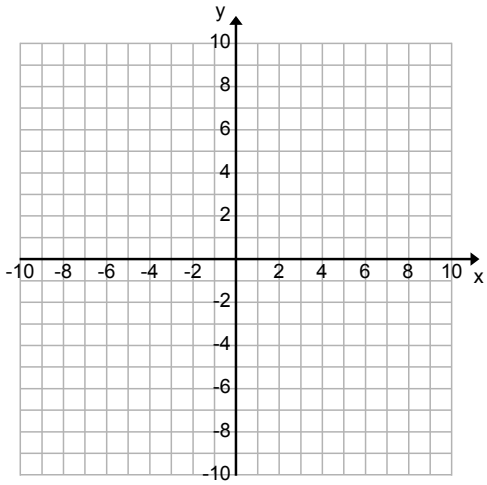
13. Each of the following functions is a translation of a "simple" function discussed in class. On the graph provided, draw a sketch (somewhat accurately) of each using translation of the basic forms.

a. (4 pts) $f(x)=\sqrt{x+4}+3$



b. (6 pts) $g(x)=\dfrac{1}{(x-4)^2}-2$

(Carefully indicate asymptotes)



14. A football stadium manager has determined that with a ticket price of $20 that an average of 20,000 people will attend the games. For each $1 increase in price, 500 fewer tickets will be sold. Let n represent the number of times the ticket price is increased by $1. Express the revenue taken in from the sale of tickets as a function of n.

15. (20 pts) Analyze and graph the following rational function using the steps mentioned in class:

$$f(x)=\frac{x^2-4}{x^2+2x-3}=\frac{(x+2)(x-2)}{(x+3)(x-1)}$$

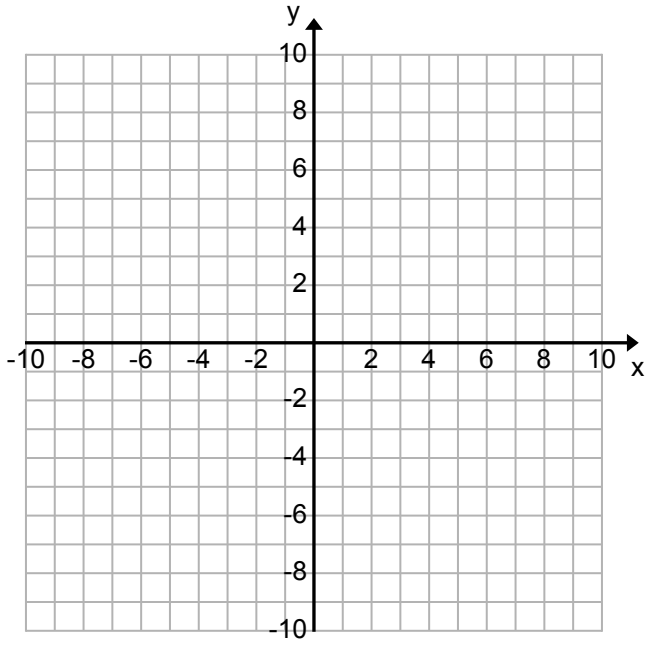
A. Find all asymptotes:

B. Find x intercepts:

C. Complete the sign graph:

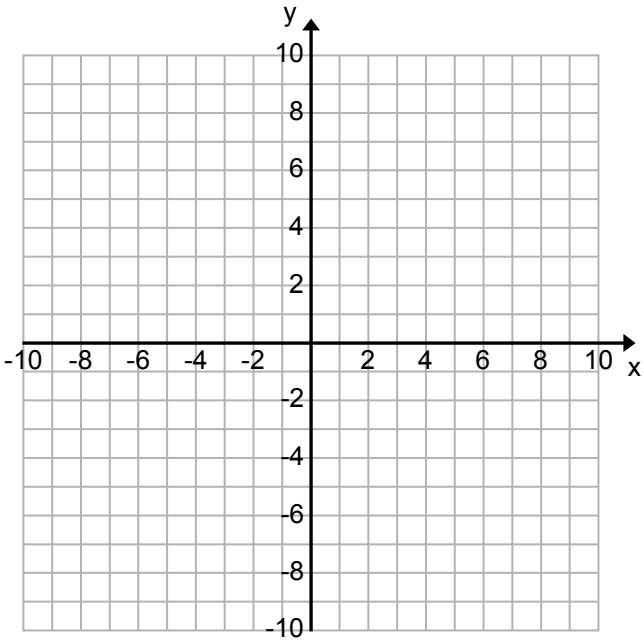
D. Determine if it crosses the horizontal asymptote. If so, state where it crosses.

E. Using the analysis above as well as any further observations, graph ***f(x)***.



16. (8 pts) Show that $\dfrac{2(x^2-4)^2-(2x-1)(x^2-4)(4x)}{(x^2-4)^4}$ is equivalent to $\dfrac{2(3x^2-2x+4)}{(x^2-4)^3}$

17. (8 pts) Graph: $f(x)=\begin{cases}2 & \text{if } x<-2\\ |x| & \text{if } -2\le x\le 2\\ -2 & \text{if } x>2\end{cases}$



Determine the domain and range of f(x). _____

18. A running track is in the shape of a rectangle with a semicircle of radius r at each end, as shown. If the total distance around the track is 200 meters, express the *area enclosed by the track as a function of r*. Do not simplify your answer but clearly define your variable(s) by using the diagram.