Sample Math Placement Exam Question solutions:

Please note that many printers will not interpret some of the notation correctly so after you print this you may want to double check some of the answers.

Algebra:

1. $x = \dfrac{8 \pm \sqrt{80}}{2} = 4 \pm 2\sqrt{5}$

2. 2x(x-4)=0 or x=0 or x=4

3. $\dfrac{(x+y)(x-y)}{x-y} \bullet \dfrac{(x+y)}{x(y+x)} = \dfrac{x+y}{x}$

4. $\dfrac{(x+2)}{(x-3)(x+2)} + \dfrac{(x-3)}{(x-5)(x-3)} = \dfrac{1}{x-3} + \dfrac{1}{x-5} = \dfrac{x-5+x-3}{(x-3)(x-5)} = \dfrac{2x-8}{(x-3)(x-5)}$

5. Let x be the time it takes them both to mow the lawn.

$$\frac{1}{3}x + \frac{1}{4}x = 1 \Rightarrow 4x + 3x = 12 \Rightarrow x = \frac{12}{7}\text{hours}$$

6. Multiplying by LCD gives the equation:
$x + 2(x-3) = 4(x+3)$ which gives $x = -18$. This answer checks in the original equation.

7. Let x be the rate of the current and t be time. The system of equations is:

$$20 = (15+x)(t)$$
$$10 = (15-x)(t)$$

Using substitution as a technique for solution, we have:

$$\frac{20}{15+x} = t \text{ so } 10 = (15-x)\left(\frac{20}{15+x}\right)$$

Solving this last equation by multiplying both sides by 15+x results in the equation:

$$10(15+x)=20(15-x) \text{ or } x = 5 \text{ mph.}$$

8. $\left(\frac{1}{9}+\frac{1}{8}\right)^{-1}=\left(\frac{17}{72}\right)^{-1}=\frac{72}{17}$

9. $\dfrac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x}+\frac{1}{y}}\bullet\frac{xy}{xy}=\frac{y-x}{y+x}$

10. $\left(\frac{1}{p}+\frac{1}{q}\right)pq=pq\Rightarrow q+p=pq\Rightarrow p=q(p-1)\Rightarrow\frac{p}{p-1}=q$

11. $\dfrac{3^{-2}x^{-12}}{5^{-2}y^{-24}z^{-4}}=\dfrac{25y^{24}z^{4}}{9x^{12}}$

12. 2{-3x+4[8+x]} = 2{x+32}=2x+64

13. 5+2x-6=2[5-4x-12] so -1+2x=-14-8x or x= $x=-13/10$

14. (1/8)(2) = 1/4

15. Let x be the length. Then 1/2 x will be the width. The perimeter is 24 so the equation would be: 2(1/2 x)+ 2(x) = 24. This gives a solution x = 8. The length is 8 inches, the width is 4 inches.

16. Using the point slope formula: $y-4=\frac{-7}{4}(x+3)$ which results in the equation: $y=\frac{-7}{4}x-\frac{5}{4}$

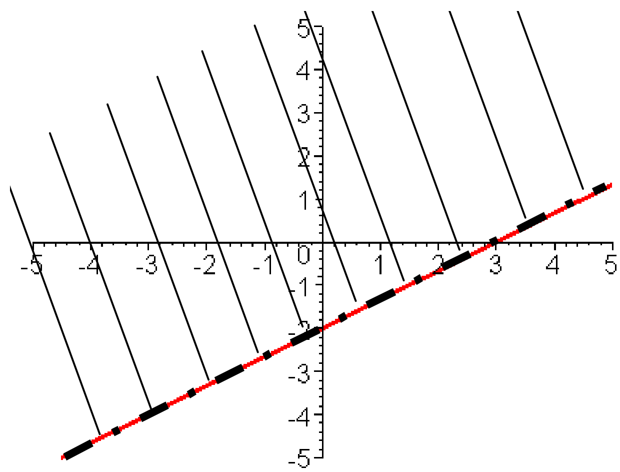
17. The slope of the given line is 2/5 so the slope of a perpendicular line would be -5/2 . Using the point (-3,1) and the point slope formula, we have: $y-1=\frac{-5}{2}(x+3)$ which reduces to $y=\frac{-5}{2}x-\frac{13}{2}$.

18. f(-3) = 2(-3)^2 -5 or f(-3) = 13.

19. Domain: {-4, -2, 3}; Range: {0,2,5}. This is not a function.

20. Using the ordered pairs: (0, .75) and (40, 4.25) and the point slope formula, we have: $W(t)=\frac{7}{80}t+.75$.

21. First graph the line 2x-3y=6 and then shade the appropriate region:



22. $-16 \le -2x \le 2 \Rightarrow 8 \ge x \ge -1$. In interval notation: [-1,8] .

23. Simplifying each side: -3x+14>2x-6 which simplifies to x < 4 . Interval notation: (-∞, 4) .

24. -3<2x+5<3 which simplifies to -4<x<-1. In interval notation: (-4, -1) .

25. Using substitution and solving the first equation for y results in :
y = -9-5x. Substituting this for y in the second equation results in :

20x+3(-9-5x) = -2.

Solving for x results in x = 5. Thus y = -9-5(5) = -34 .
The solution is thus (5, -34).

26. f(-1) is about 1 and f(2) is about 3. Thus the sum is about 4.

27. Let x be the amount invested at 6% and y be the amount invested at 8%. The two equations would be: x + y = 4500 and .06x+.08y = 307.60. Solving by

substitution results in the equation: .06x + .08 (4500 - x) = 307.60. Solving this equation for x gives x = 2620. y is then 1880. The amount invested at 6% must be $2620 and the amount invested at 8% is $1880 .

28. Solving this system by elimination, multiply the first equation by -2:

$$-4x + 6y = 4$$
$$4x - 6y = 9$$

Adding, this gives the equation 0 = 13. Thus there is no solution to the system.

29. Let d be the distance each train travels (which is the same) and t be time of the 60 mph train. We have two equations: d = 60 t and d = 90(t-3). Using substitution to solve: 60t = 90(t-3) so t = 9 hours. The distance traveled by each of the them is 60(9) or 540 miles.

30. Let x be the number of adult tickets sold and y be the number of children's tickets sold. The system of equations is

$$x + y = 548$$
$$6.5x + 3.5\,y = 2881$$

Solving by substitution: 6.5 x + 3.5 (548 - x) = 2881. The solution is x = 321. Thus there were 321 adult tickets sold and 227 children's tickets.

31. $\sqrt[3]{8 \bullet 5 \bullet x^3 \bullet x^2 \bullet y^{12} \bullet y} = 2xy^4\sqrt[3]{5x^2y}$

32. $\frac{3\sqrt{x}}{\sqrt{x}+\sqrt{5}} \bullet \frac{\sqrt{x}-\sqrt{5}}{\sqrt{x}-\sqrt{5}} = \frac{3x-3\sqrt{5x}}{x-5}$

33. $(\sqrt{x}+\sqrt{3})^2 = x+3+2\sqrt{3x}$

34. $\sqrt[4]{\frac{x^5y^{11}}{v^8}} = \frac{xy^2\sqrt[4]{xy^3}}{v^2}$

35. $5\sqrt{2}-9\sqrt{2} = -4\sqrt{2}$

36. $2x-5 \geq 0$ or $x \geq 5/2$. Domain is thus $[5/2,\infty)$.

37. $x^{\frac{2}{3}} \bullet x^{\frac{1}{2}} = x^{\frac{7}{6}} = \sqrt[6]{x^7} = x\sqrt[6]{x}$

38. Since this is a 30-60-90 degree triangle: $\frac{6}{x}=\frac{1}{2}$. Therefore x = 12 .

39. Can only simplify by factoring first: $\sqrt{9(u^2-v^2)}=3\sqrt{(u^2-v^2)}$

40. Using the Pythagorean Theorem: $12^2+8^2 = d^2$ where d is the length of the hypotenuse. This results in d = $\sqrt{208}=\sqrt{16\bullet 13}=4\sqrt{13}$ feet .

41. a. Technique of grouping: $(x+5)(x^2-2)$
b. Trinomial factoring: $(4y^2+3)(2y^2-5)$
c. Trinomial factoring: (3x-2y)(3x-5y)
d. Trinomial factoring: -1(x-7)(x+3)
e. Cubic: $5x(x-2)(x^2+2x+4)$

42. $x^2-12x+20=0$ or $(x-10)(x-2)=0$ or $x=10, x=2$

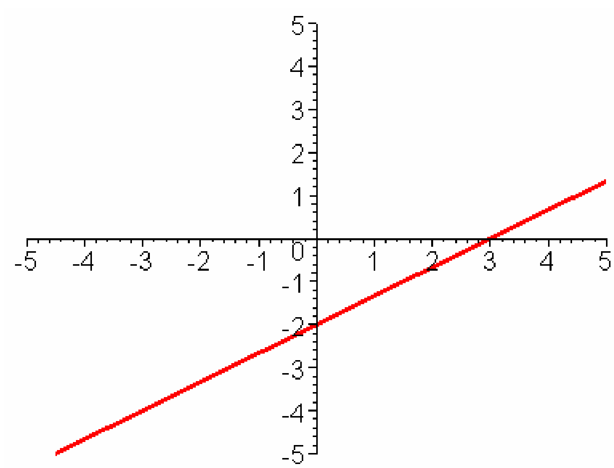
43. Let x be the width of the frame. Then the painting with the frame has a width of 10+2x and a length of 16+2x. The area information results in the equation: $(16+2x)(10+2x)=280$. Solving: x=-15 and x=2. One solution x = -15 must be ignored and the answer is x = 2 cm.

44. If one leg is x then the other leg is x+7. Using the Pythagorean Theorem:

$$x^2+(x+7)^2=13^2 \Rightarrow 2x^2+14x-120=0 \Rightarrow 2(x+12)(x-5)=0$$

The solutions are x = -12 or x = 5. The negative solution does not make sense so x = 5. Thus the longer leg is x + 5 or 12 meters.

45. This has ordered pairs (0, -2) and (3,0).



46. Using the distance formula:
$d=\sqrt{(6)^2+(-8)^2}=\sqrt{100}=10$

47. The triangles are similar since angles B and R are both right angles and angle C is congruent to angle P as given. Their sides are proportional:

6 = k(2) or k = 3.

$x = k\sqrt{5}$ so $x = 3\sqrt{5}$

$9 = k(y)$ so 9=3y so $y = 3$

END OF ALGEBRA PROBLEMS.

Precalculus Question solutions:

1. $\frac{2(x+h)^2 - 3(x+h) - (2x^2 - 3x)}{h} = \frac{4xh + 2h^2 - 3h}{h} = 4x + 2h - 3$

2. a. (-∞,∞) b. [-7, ∞) c. [-3,-.5] or [3,∞) d. x=-4.5, x=-1.5, x=.5, x=4 .
   e. (-∞, -4) or (-2,1) or (4,∞)

3. Multiplying by 2 and then simplifying: 11/2 ≤ x ≤ 7/2. In interval notation: [7/2,11/2]

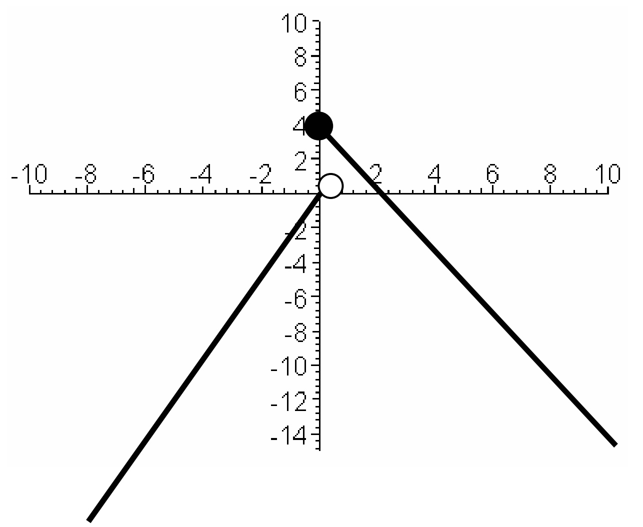
4. a. g(f(x)) = $\sqrt{x-4} - 3$ b. [4,∞)

4.5. Let c be the cost and h be the number of heaters. Two ordered pairs would be: (20, 10000) and (50,14200). The slope is then 140. Using the point slope formula: c - 10000=140(h-20) which simplifies to c=140h + 7200.

5. g(-x) is a reflection of the graph about the y axis. Thus the new x intercepts are at (4,0), (2,0), (-1, 0) and (-4, 0) .

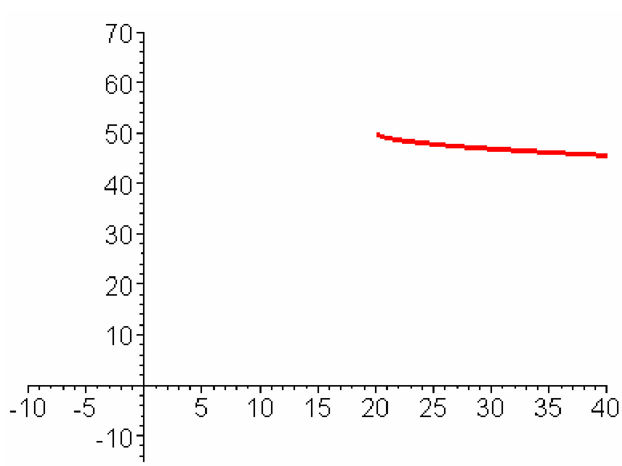
6. Possible answer: g(x) = 3x - 4 and f(x) = x^2 .

7. This is a graph of a piece-wise function:



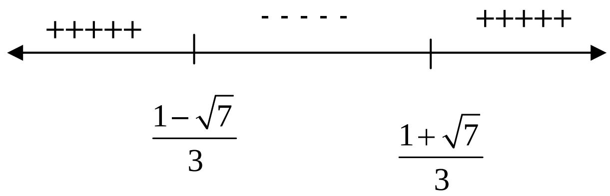
8. 3x+px = 2yx - ry or 3x + px = y(2x - r) or $y = \dfrac{3x + px}{2x - r}$

9. Reflect the graph of the square root function about the x axis, shift it 20 units right and up 50 units:



10. Range is (-∞, b].

11. Solving the equation gives the two irrational numbers: . Setting up a sign graph:



The solution is then $\left(-\infty,\frac{1-\sqrt{7}}{3}\right)\cup\left(\frac{1+\sqrt{7}}{3},\infty\right)$.

12. a. The length is given by 40 - 3x. Thus the area is A = x(40-3x).

b. The maximum can be found by determining the y coordinate of the vertex of the quadratic. Using the vertex formula, the x coordinate is 20/3. The y coordinate is then 400/3 square feet.

13. Using the formula: $y = a(x-h)^2+k$ gives $y = a(x+3)^2+6$. Since the function must also satisfy the point (1,4), the equation would be: $4 = a(1+3)^2 + 6$. Thus a = -1/8. The resulting function is then: $y=\frac{-1}{8}(x+3)^2+6$

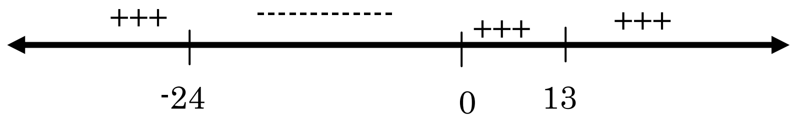
14. Substitute 30 for the height gives: $30 = -16t^2+24t+30$. Solving gives t = 0 or t = 3/2 seconds. It will be 30 feet high at the beginning of the throw and in 3/2 seconds.

15. Using synthetic division:

4⌋	3	-22	43	-12
		12	-40	12
	3	-10	3	0

Thus $(x-4)(3x^2-10x+3) = 0$ is the factored form of the equation. Using the quadratic formula for the remaining quadratic expression we get the solutions: x = 3, 1/3. The solutions to the equation are then 4, 3, 1/3 .

16. Using a sign graph:

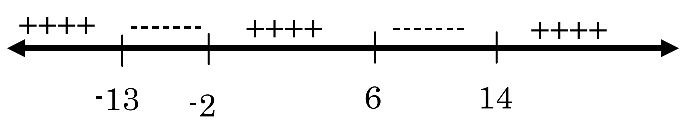


The solution is then (-∞,-24) or (0,13) or (13,∞)

17. Vertical: x = 5/3
Horizontal: y = 1/15

18. Multiplying by x^4 results in the equation: $x^4 - 7x^2 + 12 = 0$. This factors and results in : $(x^2 - 3)(x^2 - 4) = 0$. The solutions are then $x = \pm\sqrt{3}, \pm 2$.

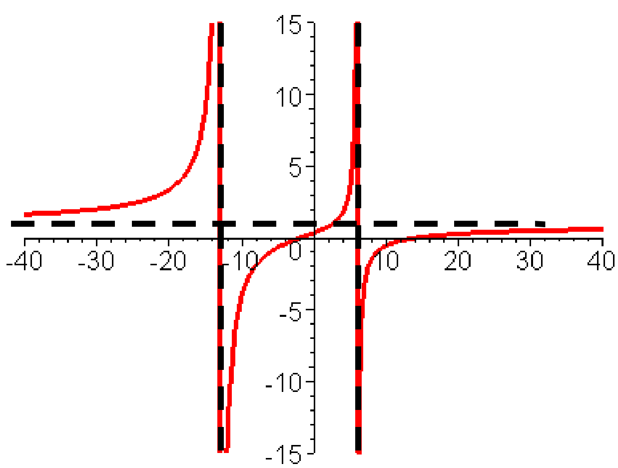
19. a. (-2,0), (14, 0), (0,14/39)
b. x = -13, x = 6, y = 1
c.



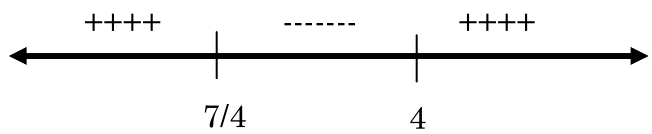
d. To determine this, solve the equation: $\dfrac{(x+2)(x-14)}{(x+13)(x-6)} = 1$.

The solution is x = 50/19 so it will cross the horizontal asymptote at the point (50/19, 1).

e.



20. $\frac{x+5}{x-4}+\frac{3(x-4)}{x-4}<0 \Rightarrow \frac{4x-7}{x-4}<0$. Using a sign graph:



The solution is (7/4, 4) .

21. Isolating the radical gives the equation: $x-4=\sqrt{2x-5}$. Squaring both sides: $x^2-8x+16=2x-5$. Simplifying: $x^2-10x+21=0$. The solutions are x = 3 and x = 7. The solution x = 3 does not check, x = 7 does check. The solution is thus x = 7.

22. Isolating the exponential portion:
$e^{2x-5}=\frac{16}{12} \Rightarrow 2x-5=\ln(4/3) \Rightarrow x=\frac{\ln(4/3)+5}{2}$

23. The surface area of each surface is:

triangles: 1/2(3)(4) = 6
base: 27
Back: 36
Slanted face: (9)(5) = 45 (Hypotenuse of triangle is 5).

The total surface area is then 2(6) + 27 + 36 + 45 = 120 square feet

24. a. x = -b/a

b. $0=\log_3(ax+b) \Rightarrow 1=ax+b \Rightarrow x=\frac{1-b}{a}$ so ((1-b)/a, 0) is x-intercept.

c. (0, $\log_3$(b))

25. $\log_3[(x+2)(x-3)]=2 \Rightarrow 9=(x+2)(x-3) \Rightarrow 0=x^2-x-15$. The solutions to this equation are then $x=\frac{1\pm\sqrt{61}}{2}$. However only the solution $x=\frac{1+\sqrt{61}}{2}$ will check.

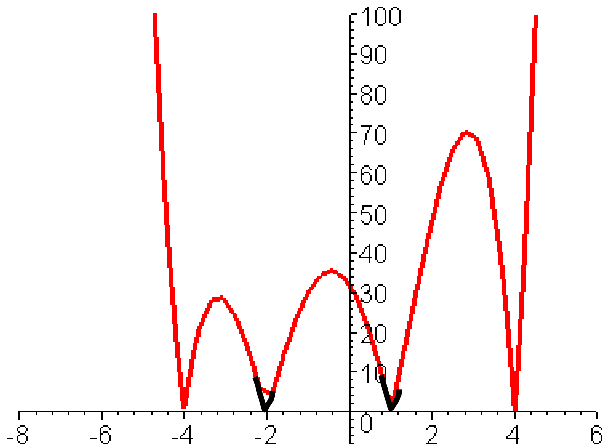
26. $\log_4(16)$ = 2 and $\log_2(16)$ = 4 . Thus 2 - 4 = -2.

27.

$x = \log_4(2y-5) \Rightarrow 4^x = 2y-5$

$$f^{-1}(x) = \frac{4^x+5}{2}$$

28.



29. Solving the inequality: $x^2 - 20x \geq 0$. Using a sign graph, we have the solution: $(-\infty, 0]$ or $[20,\infty)$.

30. Using long polynomial division, the solution is: $2y^2 + 3y - 1 + \dfrac{-1}{2y+3}$

31. Perpendicular line will have slope -1/2 since the given line has slope 2. Using the point slope formula: $y - 5 = \dfrac{-1}{2}(x+3)$ or $y = \dfrac{-1}{2}x + \dfrac{7}{2}$.

Trigonometry Question solutions:

1. Reference angle is $\pi/6$. The terminal side of the angle would be in quadrant IV.

2 $\cos\left(\frac{23\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$. Note: the trigonometric values for the six trigonometric functions for $\pi/3$, $\pi/6$, $\pi/4$ should be memorized. These are used as reference angles.

3. Using the identity: $1 + \tan^2(\theta) = \frac{1}{\cos^2\theta}$ results in $1 + 4/25 = 1/\cos^2\theta$.

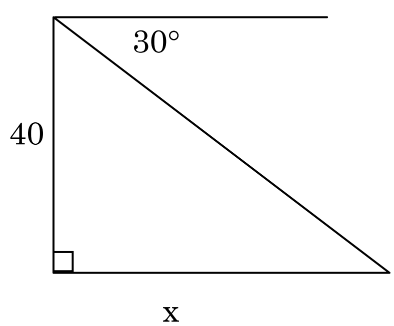
Solving for cos (θ) gives $\pm\frac{5}{\sqrt{29}}$. Since θ is in quadrant III,

$\cos(\theta) = -\frac{5}{\sqrt{29}} = \frac{-5\sqrt{29}}{29}$.

4. Using the definition of the sine function: $\sin\theta = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$

5. Reference angle is $\pi/6$ and the terminal sides of the angles must be in quadrant II and III. Thus the answers are $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$

6.



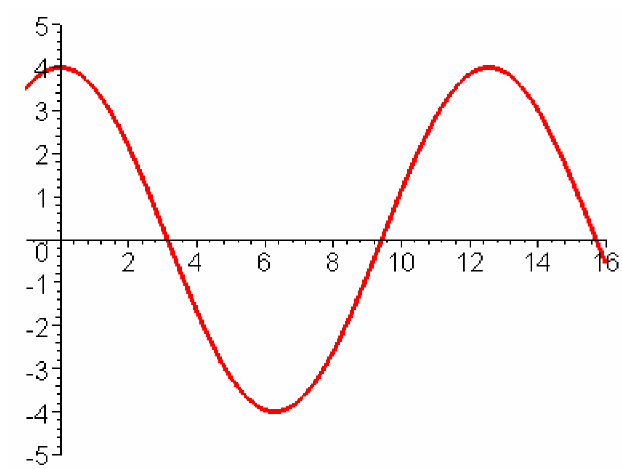
This gives the equation: $\tan(60^{\circ}) = \frac{x}{40}$. Since tan(60°) = $\sqrt{3}$, x= $40\sqrt{3}$ or about 68 meters.

7. Using an addition identity: $\sin(\frac{3\pi}{2} + x) = \sin(\frac{3\pi}{2})\cos x + \cos(\frac{3\pi}{2})\sin x$. This simplifies to -cos(x).

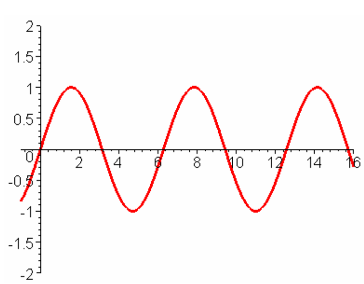
8. sin(2t)=2 sin(t) cos(t). Since sin(t) = -3/4, cos (t) = $-\frac{\sqrt{7}}{4}$ by using the Pythagorean identity. Thus sin(2t) = $2 \bullet \frac{-3}{4} \bullet \frac{-\sqrt{7}}{4} = \frac{-3\sqrt{7}}{8}$

9. $\tan\left(\frac{17\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = 1$

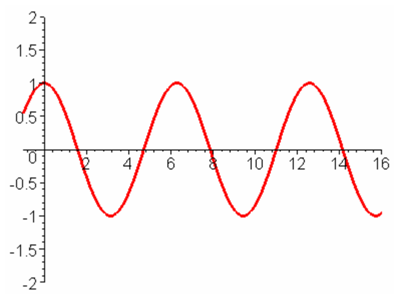
10. a. Amplitude is 4
b. period is 4π
c.



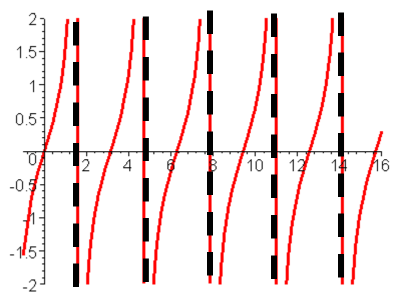
11. You should know the graphs of these three functions:
y = sinx



y = cos(x)



y = tan(x)



12. $\cos^2(x) - (1 - \cos^2(x)) = 2\cos^2(x) - 1$

13. $\sin(\theta) = \dfrac{\sqrt{9 - x^2}}{3}$