



The following final exam is intended to serve as an example only. Exam contents may change from semester to semester due to changes in faculty, textbooks, or the amount of coverage of course topics.

This is a sample M241 Exam. Each instructor may have a different format exam
Final Examination -- M241 -- Fall 1999

Name _____ Section _____

You must show all your work in a clear and orderly form to receive credit for solutions. Please remember that a primary purpose of a test is to communicate!

1. [28] Calculate the derivatives of each of the following functions:

a) $f(x) = (1 - x^3)^2 / x$

b) $g(x) = \sin(\cos(x^2))$

c) $G(x) = \int_{x^2}^3 \frac{1}{\sin(2t)} dt$

d) Compute dy/dt if $y = x^7$ and $x = 2t - 1$

2. [27] Compute the following limits:

a) $\lim_{n \rightarrow \infty} \frac{3n^2}{n(2n+1)}$

b) $\lim_{x \rightarrow 0} \frac{2 \sin(3x)}{x \cos(2x)}$

c) Determine $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ for $f(x) = \frac{a}{x}$.

3. [30] Evaluate the following integrals:

a) $\int_0^1 x^2(1+3x^3)^5 dx$

b) $\int_0^{\pi/2} \sin(2x)\cos^3(2x)dx$

c) $\int_{-2}^2 \sqrt{1-x^2} dx$ (Note: we have not developed methods for computing this integral analytically, so consider graphing the integrand as a hint for how to evaluate it.)

4. [10] If the substitution $u = x^2$ is made, the definite integral $\int_0^2 \frac{2x}{(1+x^4)^3} dx$ is equal to:

a) $\int_0^1 \frac{du}{(1+u^2)^3}$ b) 0
c) $\int_0^4 \frac{u}{(1+u^2)^3} du$ d) $\int_0^4 \frac{du}{(1+u^2)^3}$

5. [15] Each of a family of rectangles lying in the first quadrant has a diagonal joining the origin to a point on the parabola $y = 9 - x^2$. Determine the dimensions of the rectangle having largest area and justify your answer.

6. [10] Determine the arc length of $y = x^2 + 1$ from $x=1$ to $x=2$. [*You need not evaluate the integral.*]

7. [10] Determine the equation of the tangent line of the curve $y^2 + xy + x^2 = 7$ at the point $(1,2)$.

8. [10] For what value of a is $f(x) = \begin{cases} x-4, & x < -2 \\ ax^2 + 2, & -2 \leq x \end{cases}$ continuous at every x ?

Explain your reasoning.

9. [20] A 13 ft. ladder is leaning against a house when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving at the rate of 5 ft/sec.

a) How fast is the top of the ladder sliding down the wall then?

b) At what rate is the area of the triangle formed by the ladder, wall and ground changing then?

c) At what rate is the angle θ between the ladder and the ground changing then?

10. [20] You are designing a poster to contain 50 square inches of printing with margins of 4 in. each on top and bottom and 2 in. on each side. What overall dimensions will minimize the amount of paper used? Be sure to confirm that you have a minimum value.

11. [20] Graph $y = x^3 + 3x^2$. *Clearly show and label* all critical points, relative maxima and minima, inflection points and where the graph is concave up and down. Justify your conclusions analytically (i.e., not by just using the graph).

BONUS [10] Determine an equation for the straight line having slope $1/4$ that is tangent to the curve $y = \sqrt{2x-1}$.