LEARNING MATHEMATICS FROM CLASSROOM INSTRUCTION USING STANDARDS-BASED AND TRADITIONAL CURRICULA: AN ANALYSIS OF INSTRUCTIONAL TASKS

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The LieCal Project longitudinally investigates the effects of the Connected Mathematics Program (CMP) and more traditional middle school curricula (non-CMP) on students' learning of algebra. To ascertain the curricular effects, we must attend to aspects of teaching that influence students' learning opportunities. In this paper, we particularly focused on the mathematical tasks to understand the instructional experiences provided when using CMP and Non-CMP curricula. We found that teachers in CMP classrooms implemented significantly more cognitively demanding tasks than teachers in Non-CMP classrooms. Also, teachers are much more likely to encourage multiple strategies in CMP classrooms than in Non-CMP classrooms.

Purpose

One of the major goals of educational research, curriculum development, and instructional improvement is to improve students' learning. Advocates of mathematics education reform often attempt to change classroom practice, and hence, students' learning, by means of changes in curricula (NCTM, 1989; Howson, Keitel, & Kilpatrick, 1981; Senk & Thompson, 2003). Historically, curriculum has been used as a means to convey what students should learn (NCTM, 1989) and it has also been used to serve as agents for instructional improvement (Ball & Cohen, 1996). However, curriculum does not always influence classroom instruction (Ball & Cohen, 1996; Fullan & Pomfret, 1977). One of the important factors is how teachers interpret and use the curriculum materials in classroom. The purpose of this study is to examine the kinds of learning provided by classroom instruction using Standards-based and and more traditional middle school curricula mathematics curricula.

Background and Theoretical Considerations

Standards-Based Mathematics Curriculum

In the late 1980s and early 1990s, the National Council of Teachers of Mathematics (NCTM) published its *Standards* documents, which provided recommendations for reforming and improving K-12 school mathematics. In the *Standards* and related documents, the discussions of goals for mathematics education emphasize the importance of thinking, understanding, reasoning, and problem solving, with an emphasis on connections, applications, and communication (e.g., NCTM, 1989, 2000). This view stands in contrast to a more conventional view of the goals for mathematics education, which emphasizes the memorization and recitation of decontextualized facts, rules, and procedures, with the subsequent application of well-rehearsed procedures to solve routine problems.

With extensive support from the National Science Foundation, a number of *Standards*-based school mathematics curricula were developed in the United States and implemented to align with the recommendations in the *Standards* (see Senk and Thompson, 2003 or NRC, 2004 for details). The Connected Mathematics Program (CMP) is one of the *Standards*-based school mathematics curricula developed with the support of the U.S. National Science Foundation. The CMP curriculum is a complete middle-school mathematics program. The intent of CMP is to build students' understanding in the four mathematical strands of number and operation, geometry and measurement, data analysis and probability, and algebra through explorations of real-world situations and problems (Lappan et al., 2002). Because NSF-funded curricula like CMP claim to have different learning goals and also look very different from commercially developed mathematics curricula, a natural question is: What learning opportunities will a Standards-based curriculum like CMP provide that are different from the learning opportunities provided by more traditional middle school curricula?

LieCal Project

The study reported in this paper was conducted as part of a large project titled the Longitudinal Investigation of the Effect of Curriculum on Algebra Learning (LieCal Project). The LieCal Project is designed to longitudinally compare the effects of the Connected Mathematics Program (CMP) to the effects of more traditional middle school curricula (hereafter called Non-CMP curricula) on students' learning of algebra. The LieCal Project is being conducted in 16 middle schools and 10 high schools of an urban school district serving a diverse student population. At the start of the project, 27 of the 51 middle schools in the school district had adopted the CMP curriculum while the remaining 24 middle schools were using other curricula. Eight CMP schools were randomly selected from the 27 schools that had adopted the CMP curriculum. After the eight CMP schools were selected, eight Non-CMP schools were chosen based on comparable ethnicity, family incomes, accessibility of resources, and state and district test results. A total of 725 CMP students from 26 classes and a total of 698 Non-CMP students from 24 classes participated in the study, and these 1,423 students were followed for three years from grades 6 to 8 and into grade 9.

The goal of teaching is to help students learn. To understand the impact of *Standards*-based curricula, then, we must attend to aspects of teaching that appear to have potential to influence students' learning opportunities. Specifically, to help us understand the differences between the instructional experiences provided when teachers use CMP and Non-CMP curricula, in this paper we focused particularly on the instructional tasks posed and implemented by the teachers.

Instructional Tasks

Researchers have developed different paradigms and methods that can be used to identify important features of classroom instruction (e.g., see Koehler & Grouws, 1992; Porter & Brophy, 1988; Shulman, 1986). Instructional tasks have been identified as an important construct to study classroom instruction (Doyle, 1983; Stein et al., 1996). The term "instructional tasks" has been referred to by other researchers as "academic tasks," or as "mathematical tasks" (e.g., Cai & Lester, 2005; Doyle, 1983; Hiebert & Wearne, 1993; Stein et al., 1996). Mathematical tasks can be defined broadly as projects, questions, problems, constructions, applications, or exercises in which students engage. Mathematical tasks provide intellectual environments within which students can learn and develop mathematical thinking. Tasks help regulate not only students' attention to particular aspects of content, but also their ways of processing information. However, only "worthwhile problems" give students the chance to solidify and extend what they know and to stimulate mathematics learning (NCTM, 1991). In the classroom, students' actual opportunities to learn depend on the type of mathematical tasks presented and implemented. Regardless of the context, for a task to be worthwhile, it should be intriguing and it should provide a level of challenge that invites speculation and hard work. Most importantly, worthwhile mathematical tasks should direct students toward explicit learning goals by encouraging them to investigate important mathematical ideas and ways of thinking. The NCTM *Standards* (1991, 2000) recommend that students should be exposed to truly problematic tasks in classrooms so that they can practice mathematical sense making. Doyle (1988) argues that tasks with different cognitive demands are likely to induce different kinds of learning. Mathematical tasks that are truly problematic have the potential to provide the intellectual contexts for students' rich mathematical development. Such tasks can promote students' conceptual understanding, foster their ability to reason and communicate mathematically, and capture students' interests and curiosity (NCTM, 1991).

Worthwhile mathematical tasks alone do not guarantee students' learning. They are important, but not sufficient, for effective mathematics instruction because teachers may not implement worthwhile tasks as they were intended. Stein et al. (1996) found that only about 50% of the tasks that were set up to require students to apply procedures with meaningful connections were actually implemented that way. In our LieCal project, we analyzed three distinct categories of mathematical problems: those that appeared in the CMP and Non-CMP textbooks, those that were posed and implemented during classroom instruction, and those that were assigned as homework. In this paper, we report only the results from our analysis of the instructional tasks implemented in the classroom.

Methodological Considerations

CMP and Non-CMP Curricula

We have conducted detailed analyses of the CMP and Non-CMP curricula, with a focus on the algebra strand. Our preliminary analysis showed remarkable differences between the CMP and Non-CMP curricula. CMP can be characterized as a problem-based curriculum. Take the introduction to equation solving as an example. In one of the Non-CMP curricula, equation solving is introduced symbolically using the additive property (add or subtract the same quantity on both side of the equation, the equality holds) and the multiplicative property (multiple or divide a non-zero quantity on both sides of an equation, the equality holds). On the other hand, in the CMP curriculum, the introduction to equation solving is situated within real-life contexts that are used to help students understand the meaning of each step of the equation solving process (Nie, Cai, & Moyer, 2009).

The extent of the differences is also illustrated in Figure 1 below. Using a scheme developed by Stein et al. (1996), we classified the mathematical tasks in the CMP curriculum and one of the Non-CMP curricula into four increasingly demanding categories of cognition: memorization, procedures without connections, procedures with connections, and doing mathematics. As Figure 1 shows, significantly more tasks in the CMP curriculum than in the Non-CMP curriculum are higher-level tasks (procedures with connections and doing mathematics) ($\chi^2(3, \underline{N} = 3311) = 759.52$, $\underline{p} < .0001$).

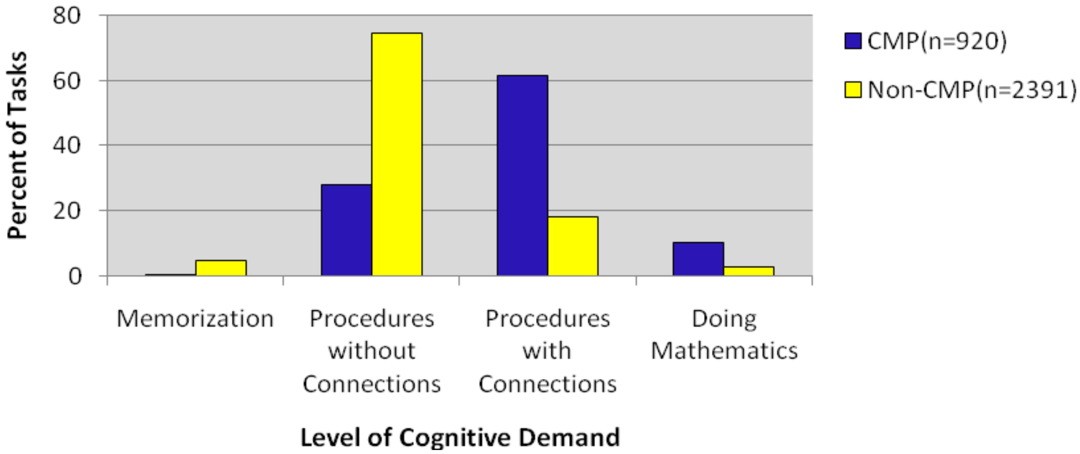

Figure 1. Percentages of various tasks in CMP and Non-CMP curricula.

Classroom Observations

As we indicated above, the research reported in this paper is part of a longitudinal study of the effect of curriculum on the algebraic thinking of approximately 1400 middle school students from 16 schools in a single urban district as they progressed from grades 6-8. The data was collected over a three-year period during 620 classroom observations. Approximately half of the observations were of teachers using the CMP curriculum. The other half were observations of teachers using Non-CMP curricula. Two retired mathematics teachers conducted and coded all the observations. The coders received extensive training that included frequent checks for reliability and validity throughout the three years. Over the course of the 6th-grade year, for example, we checked the reliability of the observers' coding three times. These three sessions revealed that the reliability of the coding done by the two specialists was quite high. The reliability achieved during the three sessions averaged 79% perfect agreement using the criterion that the observers' coded responses were considered equivalent only if they were identical (i.e., perfect match). The reliability averaged 95% using the following criteria: (a) If an item or sub-item was "scored" using an ordinal scale, then the specialists' coded responses were considered equivalent if they differed by at most one unit; (b) If an item or sub-item (e.g. representation) was "scored" by choosing from a list of alternatives all the words/phrases that characterize it, then the specialists' coded responses were considered equivalent if they had at least one choice in common (e.g. symbolic and pictorial vs. pictorial).

Each coder observed and coded about 100 algebra-related lessons each year: half in CMP classes and half in Non-CMP classes. Each class was observed four times, during two consecutive lessons in the fall and two in the spring. The coders recorded extensive information about each lesson in a 28-page project-developed observation instrument.

During each observation, the observer made a minute-by-minute record of the lessons on lined sheets. This record was used later to code the lesson. One section of the observation instrument is devoted to the analysis and coding of the mathematical tasks in the lessons. Instructional tasks were analyzed from three perspectives: (1) as intended by the author, (2) as set up by the teacher, and (3) as actually implemented by the teacher with students. The observers in the project coded each of the instructional tasks along four dimensions within each of the three perspectives: (1) Setting; (2) Solution Strategies; (3) Representations; and (4) Cognitive Demand. These dimensions are described in the results section.

Results

In this paper, we only report the results from the analysis of the tasks actually implemented by the teacher with students. In addition, the difference patterns between the tasks from CMP and Non-CMP classrooms are similar across the three middle school grade levels (6^{th}- to 8^{th} grades). Therefore, we aggregated the tasks data from all three grades. A total of 646 instructional tasks from about 300 CMP lessons and 744 tasks from about 300 Non-CMP lessons were identified.

Settings

We classified the classroom settings in which teachers implemented instructional tasks into three types: whole classroom, small group work, or individual work. The same tasks in a lesson could be implemented in different settings. Nearly 80% of the tasks in CMP lessons and 80% of the tasks in Non-CMP lessons were implemented in a whole classroom setting. About 20% of the tasks in the CMP lessons and 8% of the tasks in the Non-CMP lessons were implemented in small group settings. These two percentages are significantly different ($\underline{z} = 7.03$, $\underline{p} < .001$). On the other hand, a significantly larger percentage of the tasks in Non-CMP lessons (57%) than in CMP lessons (41%) were implemented in an individual work setting ($\underline{z} = 5.89$, $\underline{p} < .001$).

Solution Strategies

We examined whether the instructional tasks implemented in the classrooms were solved using multiple approaches or a single approach. Figure 2 below shows the percentage of tasks that were solved using multiple solution strategies and the percentage of tasks that were solved using a single solution strategy in both CMP and Non-CMP classrooms. A Chi-square test shows that while a larger percentage of the tasks implemented in CMP classroom were solved using multiple solution strategies, a larger percentage of the tasks implemented in Non-CMP classrooms were solved using a single solution strategy ($\chi^2(1, \underline{N} = 1390) = 122.49$, $\underline{p} < .0001$).

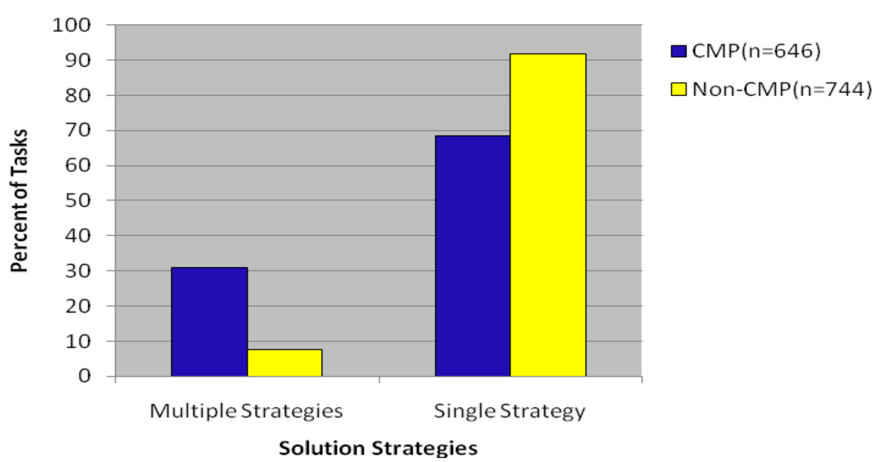

Figure 2. The distribution of solution strategies in CMP and Non-CMP classrooms.

Representations

The representations used to solve each problem were classified into 7 categories: (1) symbolic, (2) written words, (3) pictorial, (4) tabular, (5) graphical, (6) verbal, and (7) physical manipulatives. Table 1 below shows the percentage of tasks implemented in CMP and Non-CMP lessons using each of the representations. The solution to an implemented task can involve multiple representations. Only a small proportion of the tasks implemented in CMP and Non-CMP lessons were represented with physical manipulatives. The most frequently used representations of implemented tasks in both the CMP and Non-CMP lessons were symbolic, and the proportion of the tasks that were represented using symbolic representations in Non-CMP lessons was greater than that in CMP lessons (z = 6.16, p < .001). Compared to the use of symbolic representations, the proportion of using other representations is much smaller in both CMP and Non-CMP lessons. We compared the frequencies with which written words, pictorial, tabular, graphical, and verbal were used to represent implemented tasks in both CMP and Non-CMP lessons. We found that the proportion of the instructional tasks that were represented using each of these representations (written words, pictorial, tabular, graphical, or verbal) in CMP lessons was greater than that in Non-CMP lessons (z = 3.80 – 8.78, p < .001).

Table 1. *Percentages of Tasks With Each of the Representations*

	Symbolic	Written Words	Pictorial	Tabular	Graph	Verbal	Physical Manipulatives
CMP (n=646)	78	20	30	26	23	20	4
Non-CMP (n=744)	90	12	15	8	8	6	3

Cognitive Demand

Using a scheme developed by Stein et al. (1996), we also classified the instructional tasks from CMP and Non-CMP classrooms into four increasingly demanding categories of cognition: memorization, procedures without connections, procedures with connections, and doing mathematics.

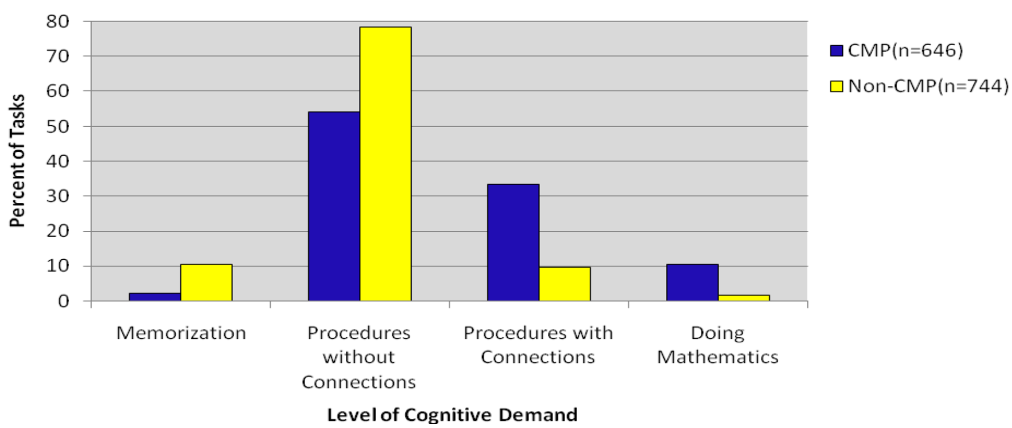

Figure 3. Instructional tasks implemented in CMP and Non-CMP classrooms.

Figure 3 illustrates the percentage distributions of the cognitive demand of the instructional tasks implemented in CMP and Non-CMP classrooms. A chi-square test shows that the CMP and

Non-CMP percentage distributions are significantly different ($\chi^2(3, \underline{N} = 1390) = 209.42$, $\underline{p} < .0001$). The difference is due to the fact that there was a larger percentage of high cognitive demand tasks (procedures with connection or doing mathematics) implemented in CMP classrooms than in Non-CMP classrooms ($\underline{z} = 13.79$, $\underline{p} < .001$). On the other hand, there was a larger percentage of low cognitive demand tasks (procedures without connection or memorization) implemented in Non-CMP classrooms than in CMP classrooms.

Discussion

The research reported in this paper is part of a larger longitudinal study conducted in the LieCal Project. The LieCal project was designed to provide (1) A profile of the intended treatment of algebra in the CMP curriculum with a contrasting profile of the intended treatment of algebra in the Non-CMP curricula; (2) a profile of classroom experiences that CMP students and teachers undergo, with a contrasting profile of experiences in Non-CMP classrooms; and (3) a profile of student performance resulting from the use of the CMP curriculum, with a contrasting profile of student performance resulting from the use of Non-CMP curricula. In this paper, we analyzed a single aspect of the classroom experiences that CMP and Non-CMP students and teachers underwent, namely the instructional tasks implemented in the two types of classrooms. The initial analysis of the implemented instructional tasks clearly showed remarkable differences between CMP and Non-CMP classroom instruction. Instructional tasks are more likely to be implemented in small group settings in CMP classrooms than in Non-CMP classrooms, and vice versa for the tasks implemented in individual settings. The instructional tasks implemented in CMP classrooms were more than three times likely to be solved using multiple solution strategies than they were in Non-CMP classrooms. While solutions of instructional tasks in Non-CMP classrooms were more likely to be represented using symbols, solutions of instructional tasks in CMP classrooms were more likely to be represented using written words, pictorial representations, graphs, tables, or verbal representations. In addition, CMP teachers were more than three times as likely to implement high-level tasks during classroom instruction than Non-CMP teachers.

The findings of this study not only show the importance of examining the instructional experiences of students using CMP and Non-CMP curricula, but it also shows the power of focusing on instructional tasks to reveal the instructional differences. Recall that our analysis of the mathematical problems in the CMP and Non-CMP curricula showed that significantly more tasks in the CMP curriculum than in the Non-CMP curriculum are high level tasks (procedures with connections and doing mathematics). Thus it is reasonable to infer that the differences in setting, strategy, representation, and cognitive level of the tasks implemented in CMP and Non-CMP classrooms reflect the differences between the mathematical problems in the CMP and Non-CMP curricula.

The striking and clear differences between CMP and Non-CMP classrooms are of great interest and importance in our longitudinal investigation of the impact of curriculum on students' learning. As part of the parent study, we also collected large-scale, longitudinal student achievement data. In our presentation, we will identify and present important linkages between students' classroom experiences and their learning outcomes.

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