1. Determine if the following two series converge or diverge. Please give a justification in each case.

\[ \sum_{n=2}^{\infty} \frac{1}{n \ln(n)}, \quad \sum_{n=1}^{\infty} (-1)^n \cos(1/n). \]

The first one is \#21 in 11.3. It is similar to Example 4 on p. 700. The terms \( \frac{1}{n \ln(n)} \) decrease to zero. Since the integral

\[ \int_{2}^{\infty} \frac{1}{x \ln(x)} \, dx \]

diverges. The series diverges by integral test.

In one section, the problem is on \( \sum \sin(1/n) \). This is \#31 in 11.4. Use limit comparison test with the harmonic series \( \sum \frac{1}{n} \).

The last one here diverges since \( \cos(1/n) \) goes to 1. The series diverges by \( n^{th} \) term test.

2. Use the first 100 terms to approximate the series \( \sum \frac{1}{n^3 + 1} \). Estimate the error involved in the approximation.

This is Example 5 on p.708, one that I asked the class to over.

3. Under what conditions does an alternating series converge?

Use the stated conditions to verify that following series converges.

\[ \sum_{n=1}^{\infty} (-1)^n \frac{n}{e^n}. \]
The conditions have been discussed in class and in text 11.5. The function \( f(x) = xe^{-x} \) decreases for \( x \geq 3 \) and decreases to 0 by L'Hopital's rule.

4. (a) Give an estimate of the positive sum \( \sum_{n=1}^{10000} \frac{1}{\sqrt{n}} \).

This is a variation of Example 6. The sum is less than \( \int_{1}^{10000} \frac{1}{\sqrt{x}} \, dx \) according to estimate 4 on p.703. I did a couple of examples in class.

(b) Let \( S = \sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}} \). If \( \sum_{n=1}^{9999} (-1)^n \frac{1}{\sqrt{n}} \) is used to approximate the alternating series \( S \), estimate the error.

The error is estimated by the \( N + 1^{st} \) term.