Notice: This is a sample exam provided for practice for students in these two sections 080 and 081. It should not be used by other groups especially commercial sites outside the University.

MATH 242 Final Exam SPRING 2007 Leung

Name: ________________________________ Section Time: 8 10am circle one

- Please read each question carefully.
- Please show your work clearly on the sheets provided.
- Answers without justification will be assigned zero credits. Explanation provided after the exam will be ignored.
- Each of the problems in the exam has appeared in class/text or previous tests. Do not indulge yourself on a particular problem for too long.
- It is your responsibility to check that you have SEVEN different problems in your exam.
- Keep your eyes on your own exam.

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I. a) Write down the Maclaurin series for $e^x$ at $a = 0$. You need to write down the general $nth$ term of the series. If you cannot remember the formula, you may derive the coefficients again by computing the derivatives first.

b) Using appropriate methods discussed in class, find all the values of $x$ that Maclaurin series converges at.

c) Using the result in a) or by direct computation, find the first four non-zero terms of the Maclaurin series for $e^{-x^2}$.

d) If the value of $\int_0^1 e^{-x^2} \, dx$ is approximated by the integral of the answer in part c), give an estimate on the error.
II. a) Find the third degree Taylor polynomial of \( f(x) = \sqrt{x} \) at \( a = 4 \). Give a reasonable approximation of \( \sqrt{3.8} \) using your answer. Give an estimate of the error.

b) A certain ball has the property that each time it falls from a height \( h \) onto a hard surface, it rebounds to a height of \( h/2 \). Suppose that the ball is dropped from a height of 20 meters, find the theoretical total distance travelled by the ball assuming it would bounce indefinitely.
III. The polar graph \( r = 1 + \sin(\theta) \) is called a cardioid.

a) Check if the graph has symmetries with respect to both coordinate axes by using the methods discussed in class/text.

b) Plot the graph of the polar equation by setting up a table of appropriate values of \( \theta \) and \( r \).

c) Find the area of all the regions INSIDE the polar curve \( r = 1 + \sin(\theta) \) and OUTSIDE the circle \( r = 1/2 \). You may use the identity: \( \cos^2(x) \equiv (1 + \cos(2x))/2 \).
IV. Given the infinite series
\[ \sum_{n=1}^{\infty} ne^{-n}, \]
use the ratio test AND integral test to prove the convergence of this series. (You need to show both methods here)

By differentiating the geometric series for \( \frac{1}{1-x} \), can you find the exact value of
\[ \sum_{n=1}^{\infty} ne^{-n}? \]
V. Evaluate the following integrals. Convert the answer in a) in terms of \( x \).

\[
a) \int \frac{x^3}{1 + x^2} \, dx, \quad b) \int_0^1 \sin^{-1}(x) \, dx.
\]
VI. A particle travels along a parametric curve given by $x = 2 \cos(t)$, $y = \sin(t)$ for $t \geq 0$.

Show that this curve is one of the conic sections.

Give a rough sketch of this graph for $0 \leq t < 2\pi$ and compute the foci of this curve.

Set up an integral formula for the surface area generated when this curve is rotated about the x-axis. be careful with the set-up.
VII. i) If \( P = P(t) \) satisfies the equation \( \frac{dP}{dt} = 10 - 2P \), with \( P(0) = 3 \), solve for \( P(t) \) explicitly.

ii) Find by any method covered in class,

\[ \lim_{t \to \infty} P(t). \]