1 Introduction

Many species of animals self-organize into coherent groups. The purpose of these groups is to enhance the fitness of the individuals within them in a variety of ways. Aggregation provides lone animal’s greater survivorship, and reproductive success. [3] Remarkably, these groups form without any sort of central control. Animals behavior in a social setting has been researched in the past. Large scale group interactions are all around us. Everything from birds migrating south for the winter, to crowds walking down the streets of New York depends on group dynamics. In today’s society group interactions can also be seen on a more technological level. Video games, autonomous machines, animated movies and robotics are a few markets where individual and group behaviors are studied. If groups of animals can be modeled more accurately it could be very valuable to society, as well as to the environment. Streets and roadways may be designed to allow more efficient travel for humans. If we know how animals move and migrate, we may be able to better protect them and their habits from deforestation and pollution, by avoiding the areas which they will be traveling through, and living in. The implications of having such knowledge of group interactions can have an impact into many areas of our lives.

Our project will gain knowledge through modeling group behaviors, as seen in nature, by using a computer matrix laboratory program called Matlab. Matlab will be a vital tool for our project; it will give us a way to show how our model will move in a simulated 3-D environment. We will primarily be limiting ourselves to looking at and modeling the behaviors of fish and fish schools. ”Simple local interactions between individuals can produce complex adaptive patterns at the level of the group” [4]. If we can discover why and how these interactions take place we can better model their natural behavior. There are many other factors that influence the behavior of the fish swarms, which we will need to better understand before we can successfully produce an accurate model. For example, we know that a small proportion of individuals that are informed are required to achieve great accuracy of movement. The larger the group, the smaller proportion of informed individuals needed to guide the group [1]. What we don’t know however is what that proportion of leaders is or how these leaders interact and pass information to the other members of the swarm. There has been a lot of work done in this field relating to a three zone theory. [2] This theory gives each animal a zone of repulsion, orientation, and
attraction. We believe that fish use these zone interactions as well as a set of other rules to control their behavior in relation to those around them. We hope to build upon what is already known about the behaviors of fish and fish swarms by creating our own swarming algorithm based upon previous research done in the field as well as providing our own insights into what we think realistic behavior for fish actually is. The major limitation we have to this approach is that there has not been a lot of observation made of individual fish within fish swarms in their natural environments.

Groupings of artificial life are mirrored in our everyday lives. In this project we are simulating natural aggregates. We chose schools of fish, but this type of aggregation goes beyond what we are studying and relates to our everyday lives. Behavior between different individuals can be seen in nature everywhere, but in todays society it can be seen on a more technological level. Video games, autonomous machines, animated movies and robotics are a few markets where individual and group behaviors are studied. We will find a mathematical model for the behavior of fish in schools. We are trying to understand the natural interactions between individuals within the school. Throughout nature we see individuals with knowledge and experience leading the way for others. An example of this would be ants finding food. As one individual finds a food source, the others are told and then eventually there are a number of ants benefiting from what one ant found. We will simulate migrating fish through educating leaders of the school who will then influence the paths of the other individuals.

2 Assumptions

Though there are many species of animals that exhibit swarm behavior we feel that our model most accurately describes the behavior of fish. With fish, we do not have to worry about gravity, wind, or changing land elevations. We only assumed that there was constant water resistance. Each fish is given a field of vision of 150 degrees. We are assuming that all fish interact with one another, and respond to changes in the environment in the same way. Because there is evidence that fish behavior is not random, by creating a mathematical mode we will be able to simulate the swarm behavior. For example, we use position and velocity equations to model what each fish is going to do in the next time step.

3 Model

3.1 Milling behavior in a swarm

The first challenge we had to face was deciding how a group of fish would react given no external stimulus. After considering everything that we had learned about fish behavior to this point we decided that the fish should swarm in a torus formation. This milling formation is the behavior the fish will always return to when they are being acted on by another force, for example a desire to eat or being pursued by a predator.
We read in an article, by Viscido, that fish have a blind spot and they have vision of 150 degrees. With this new vision, the fish will only see what is in front of him, so he now follows the closest one in front of him. This made our model more realistic because before this vision tunnel, the fish were bouncing back and forth not knowing which individual to follow.

3.2 Traveling as a swarm to one point

A principle property of animal swarm behavior is moving from point to point. This characteristic needs to be incorporated into our model so that we can more accurately model overall behavior. When a swarm travels as a group to another point they do so with a minimum number of leaders. Once the swarm travels to the destination they will resume their normal milling behavior. In the model, we specify the number of leaders, which know the final destination. The leader fish will turn away if another fish is in his zone of repulsion. The non-leaders pick one fish to follow, unless another fish enters their zone of attraction and is more than 25 percent closer than the current fish that is being followed. The fish also have a zone of repulsion. If a fish is too close to them they will turn away from it.

Each leader knows exactly where the fixed point is and goes directly towards it. The leaders direction is calculated by subtracting the leaders position from the point and normalizing. The equation:

\[ \vec{d}_1 = (1 - \lambda)\vec{d}_0 + \lambda\vec{d}_a \]  

For Leaders: 

\[ \vec{d}_a = \frac{\vec{x}_{i-1} - \vec{f}}{|\vec{x}_{i-1} - \vec{f}|} \]  

Where \( d_0 \) is the fish’s directional vector at the previous timestep, \( x_{i-1} \) is the leaders current position and \( f \) is the desired endpoint of the leaders. For others:

\[ \vec{d}_a = \frac{\vec{x}_{i-1} - \vec{x}_l}{|\vec{x}_{i-1} - \vec{x}_l|} \]  

Where \( x_l \) is the position of the closest fish in the zone of attraction. If another fish is in the zone of repulsion the following equation describes the direction vector of the fish.

\[ \vec{d}_a = -\frac{\vec{x}_{i-1} - \vec{x}_l}{|\vec{x}_{i-1} - \vec{x}_l|} \]  

The value \( \lambda \) limits how fast the fish can change direction. This is done by using part of the new and old direction vectors. This helps to have the movements be more natural. The value for \( d_a \) is different between the leaders and followers. For the leaders, their normalized direction vector is always towards their final destination, while the other fish will follow the closest fish in their field of vision. However, all fish, including the leaders will follow the
repulsion equation, if there is a fish in their zone of repulsion. Once $d_1$ is calculated it is multiplied by the constant scalar velocity:

$$\vec{V}_1 = V * \vec{d}_1$$

(5)

This is then used to describe the velocity of the fish during the next time step. Currently, we set $\lambda$ to .8 so that it mainly follows its new direction vector. The model has 50 different fish in the viewing area and has the ability to vary the number of leaders as well as the swarm size.

### 3.3 Aversion of a Predator

Another prominent property of animal behavior is the aversion of a predator. To implement this we have the fish repel from the predator when they are within the zone of predation. This value is set to be the same size as the zone of attraction. To do this we used the equation:

$$\vec{d}_b = \frac{1}{x_i - \vec{P}}$$

(6)

In this equation the fish are repelled directly away from the predator to avoid getting eaten. With the addition of this condition the controlling algorithm will be modified to be:

$$\vec{d}_1 = (1 - \lambda)\vec{d}_0 + \lambda(\vec{d}_a + \vec{d}_b)$$

(7)

Since the zone of predation is the same size as the zone of attraction the fish will always have some desire to avoid the predator. However there desire to move away from the predator does not get large until the predator is close.

### 4 Analysis/Results

#### 4.1 Traveling as a swarm

##### 4.1.1 Testing of Model

In order to find the optimum amount of leaders needed to direct the swarm there is a need to experiment with the model. From the Couzin paper on leadership it says that around 5 percent of the swarm are usually informed leaders. We varied both the swarm size and number of leaders. The range for the number of leaders has been between 2-10 percent and the swarm size has ranged from 20-50 individuals. To show which tests work better then others we have calculated the distance of the center of mass from the desired location. [1]

$$R_{cm} = \sum_{i=1}^{N} \frac{m_i R_i}{m_i}$$

(8)

Since the mass of all of the fish is the same the equation simplifies to:

$$R_{cm} = \frac{\sum_{i=1}^{N} R_i}{N}$$

(9)
In this equation $R_{cm}$ is the position of the center of mass with respect to the origin and $m_i$ and $R_i$ are the mass and position of the fish respectively.

### 4.1.2 Results

For the analysis of the data we performed various tests while varying both size of swarm and percentage of leaders. For the tests we needed to keep some values the same. In all of our tests, we used the point $(5,5,5)$ as the desired final location. The area that the fish are randomly placed in is a 10x10x10 cube. There are also five tests performed and averaged per case so that there will be a more accurate representation of what happens. In the first test that we performed, we kept the percentage of leaders constant and varied the number of fish in the swarm. The reason for this test is that in Couzin’s article “Effective leadership and organization of swarms on the move,” he stated that the number of leaders necessary in the swarm is a constant percentage throughout various swarm sizes.

![Figure 1: Data from test performed with 10 percent leaders in a swarm size varying from 20 to 110](image)

In this trial, shown in figure 1, the smaller swarms converge quicker with same percentage of leaders. We performed the test for 1000 time steps. In this model one time step is equivalent to .1 seconds. In the real large swarm size, the swarm had not converged after this range of time. This led us to believe that the percentage of leaders is not a key factor in determining the optimum amount of leaders in a swarm. We concluded that there was more testing to be done. The next test that we performed was when we varied the amount of leaders in the swarms of different sizes.
Figure 2: Data from test performed with 5 leaders in a swarm size varying from 20 to 110

In figure 2, we conducted a test with a constant 5 leaders among group sizes ranging from 30 to 110. From the graph, you can see that when the group size is larger the small percentage of leaders does not effectively make the fish follow to one point. This data proves that when the number of leaders is small, the group does converge in an efficient manner.

The next graph, Figure 3, shows the varying percentage of fish among a group size of 50. In this case, there is milling at the extreme value of zero leaders because the swarm has no desire to move from its current location. The test with 50 leaders was also performed so that the convergence ability was only based on the speed that the fish were traveling. There were also tests done at medium ranges between these extremes.

In Figure 4, the time it took for the swarm to converge to a minimum value with a range of leaders is shown. The time to converge was estimated as the first observed local minimum. This graph effectively shows that the increasing amount of leaders allows for a quicker convergence of the swarm. However, with this graph the case where there are no leaders was not included because the swarm never converged.

To choose the best scenario for the number of leaders in a swarm size of fifty the added value was calculated. In accordance with the economic principle of the Law of Diminishing Return, the decrease in time to converge was divided by the additional leaders added. Because of the limited number of trials performed the cases of 7 and 9 leaders produced
Figure 3: Data from test performed with a constant swarm size of 100 and leaders varying from 0 to 100 percent.

Figure 4: Graph of the time it took the swarm to first reach its minimum values with varying a varying number of leaders.

Inconsistent results and were not included in Figure 5. From this graph, we chose 5 leaders for the optimum number of leaders for a group size of fifty.
4.2 Avoiding a Predator

4.2.1 Testing

To test the effectiveness of our predator aversion we first performed a test with the predator not destroying them. The purpose of this was to see how the swarm would behave while being chased and disturbed from their normal behavior. The other test that we conducted is the time it took the predator to eat all of the fish at various points. The predator has a constant starting point of (-5,-5,-5). The purpose of having the fish start at various points is to simulate various attacks by a predator.

4.2.2 Results

Figure 6 shows the distance of the center of mass of the swarm from the desired point while being chased by the predator. After being chased most of the swarm sizes remain at a similar range from the desired point.
In Figure 7, various attacks by the predator are simulated. The time to eat the entire swarm is varying by attack. The first test is when the swarm is trying to get to the point (5, 5, 5). When the predator attacks from (-5, -5, -5) he is able to eat the swarm rather quickly because he is in their blind spot. It actually takes longer to eat the swarm when they are swimming at him (towards point (-5, -5, -5)). We also conducted a "worst case scenario" where the swarm stays at the origin and the predator attacks them. This proved to be the quickest time to eat the entire swarm. Finally, tests were also done at further distances of (10, 10, 10) and (-10, -10, -10). These results are similar to those at closer distances in that the swarm gets eaten quicker when they are trying to get to the point (10, 10, 10).

Figure 6: Graph of distance of the COM of the swarm from the desired point while being chased by a predator
4.3 Effect of Sight Cone angle on swarm behavior

4.3.1 Testing

When all animals travel the have limited vision due to the range of vision that they possess. This varies from species to species depending on multiple factors. To test the effect of the sight cone angle on a swarm a test was done by varying the sight angle of the fish from 15 to 360 degrees. There is a constant five leaders in a swarm size of fifty in every test. Even though we area using 150 degrees for the rest of our model it is interesting to see the effect that this has on the swarm.

4.3.2 Results

From the testing we found that, as expected the animals with the larger sight cone have a greater ability to move to a point quicker because they can see all of the other fish around them. However, as you can see in figure 8 there are some irregularities, in that the smallest sight cone didn’t do the worst and the 270 degree sight cone had the best performance. This however, is most likely due to the fact that there were not enough tests performed to block out the fluke data.
5 Strengths and Weaknesses

5.1 Strengths

Our model works with many different leaders and swarm size. Even with a small amount of leaders the swarm is able to perform effectively. The smallest leader to fish ratio that we attempted was one leader for fifty fish. This model still worked after time, although it took over a minute for all of the fish to converge on the target area. We feel this model is the most natural and most realistic model possible for the given milestone. The leaders are not known by other fish, they mill around the desired point, the model works with different swarm sizes and number of leaders. In the implementation of the predator into the milestone the swarm still acts in a natural manor. The fish can only detect the predator if it is within their field of vision. Also, swarm scatters as predator gets closer to the group. Finally, to simulate the worst case scenario conditions we used the original "Super Predator" which eats as many fish as possible at a time and doesn’t take any time to chew them.

5.2 Weaknesses

When the leader arrives at the final destination the other fish are not quite able to get all the way to the final destination because of our zone of repulsion. When the leaders arrive at the final destination, they exhibit unnatural behaviors. They turn quickly back and forth until another fish enters the leaders zone of repulsion and moves him out of the direct center. When we have the destination located at the outer limits of our plot, the fish seem to form strings, which make it look as if each fish is following a specific leader. We have
them following the fish that is closest in their zone of attraction, but we will have to make this more random because real fish are erratic. The fish on the outside of the milling pattern do not reach the actual target because the zone of repulsion doesn't allow them to. In the implementation of our model we have encountered some areas to improve. First is the fish have no way of letting other fish know a predator has been spotted. The leaders get eaten quicker because they will not follow someone who is avoiding a predator. Instead they attempt to get to the destination at all costs. Finally, the fish have no memory. If they turn away from the predator, as soon as he is out of their field of vision he no longer exists to them.

6 Summary and Conclusion

Individual behavior and interactions among gregarious fish form groups that respond to individual movements and other impulses. We modeled them through the knowledge we gained from reading. Through our work we were able to simulate milling, traveling to a certain point, and predator aversion. Milling was simulated by incorporating blind zones and initial position. This was done by creating leaders which traveled around a certain point. Through the implementation of blind spots and zones of attraction and repulsion we were able to simulate this milling behavior. The next behavior that we modeled was a swarm traveling to a fixed point. To do this we created leaders, which traveled to a stationary point. The leaders were then followed by the other individuals, which then milled around the given point. We non-dimensionalized so that we could implement a similar behavior to the rest of the swarm. The final challenge was to model the predator aversion behavior in swarms. This was done by adding another parameter to our algorithm, which had the fish wanting to move away from the predator when he got too close to them. Finally, we were able to see the effects of the sight cone on the behavior of a swarm. This is important because there are many different species with different sight limitations. Through all of our work we have created an effective model that simulates a wide variety of swarm behavior.

7 Appendix

References

