Fifth Lab For M242, Summer 2000

The lab is due on Tuesday, June 27, 2000. You may have oral discussions with any one in this section but the Maple work must be your own. Absolutely no work is to be shared. Please be warned, all parties to any violation of this rule will be held equally responsible and will get an **F** in this course.

Topic : Using procedures

In Maple, so far we have used expressions instead of functions. For example, if we want to work with the function $x^3 + \sin x$ then we have chosen to use

```
p := x^2 + sin(x) ;
```

However, the `p` defined above is an expression and not a function. If we want the value of that expression at some point then we use the `eval` command. A better way may be to define a Maple function or procedure as

```
f := proc(x) x^2 + sin(x) end ;
```

Every Maple procedure has the form

```
name of procedure := proc( variables separated by commas )
                          value of the function
                      end ;
```

So next is an example of a procedure dependent on two variables

```
g := proc(x,a)
     x^3 + a^3 + x*sin(a) ;
     end;
```

Examine the following use of the procedures f and g carefully and make sure that the output of each command is what you expect it to be.

```
f(2);     exact value of f at x=2
f(2.0) ;   numerical value of f(2)
fp := D(f);  fp is the derivative of f - it is also a procedure.
D(f)(3);  exact value of derivative of f at x=3
D(f)(3.0); numerical value of derivative of f at x=3
plot(f, -2..2);  graph of f over the interval [-2,2]
g(2,1);
g(2.0, 1.0);
```

Procedures are useful in implementing algorithms.

1. [10]

   (a) Define a procedure named `f` whose value at x is $x^4 - 2x^3 - x^2 - 2x + 2$.

   (b) Plot the graph of f over the interval $[-1, 3]$. How many roots of $x^4 - 2x^3 - x^2 - 2x + 2 = 0$ are there in the interval $[-1, 3]$.

   (c) By zooming in (change the interval over which the graph is plotted), determine the left root, accurate to four places after the decimal. Just show the last graph you obtain, do not print the earlier graphs.

   (d) Evaluate f at this value - how close is it to zero?

2. [10] We will now determine this root using the bisection method. This method is based on the principle that if $f(a)$ and $f(b)$ have opposite signs then $f(x) = 0$ has one root in $[a, b]$. So the algorithm is the following

   - Using the graph determine an interval $[a, b]$ so that $f(a)$ and $f(b)$ have the opposite sign. Assign these values to a and b.
   - Determine m the midpoint of the interval.
   - If $f(a)$ and $f(m)$ have opposite signs then root is in $[a, m]$ so set $b := m;$, otherwise root is in $[m, b]$ and so set $a := m;$.
   - If the length of the interval $[a, b]$ is small enough then stop, else go back to step two.

   Using the bisection method, find the left root of $f(x) = 0$ accurate to two places after the decimal. Please start with $a = 0$, $b = 1$.

3. [10]

   (a) Using Newton's method, determine the left root of $f(x) = 0$ accurate to eight places after the decimal. Be careful about your starting guess.

   (b) Using Newton's method, determine the right root of $f(x) = 0$ accurate to eight places after the decimal. Notice how fast this is compared to the bisection method.

   (c) What are the values of f at these approximations to the roots.

What To Hand In

Submit a printout of your Maple worksheet - stapled together. The report must be organized as explained in section 2.1 of the tutorial.

Rakesh