Consider the following equation:

$$\epsilon y'' - y = 0, \quad y(0) = 1, \quad y(1) = 1.$$  

(1)

Its exact and uniform asymptotic solutions are given by

$$y(x) = \cosh\left(\frac{x}{\sqrt{\epsilon}}\right) + \left[1 - \cosh\left(\frac{x}{\sqrt{\epsilon}}\right)\right] \frac{\sinh(x/\sqrt{\epsilon})}{\sinh(1/\sqrt{\epsilon})},$$

(2a)

$$y_u(x) \sim e^{-x/\sqrt{\epsilon}} + e^{(x-1)/\sqrt{\epsilon}}.$$  

(2b)

The exact solution is graphed below.

Solution of (1) [as given by (2a)] with $\epsilon = 5 \times 10^{-4}$. 
Consider the following equation:

$$\epsilon y'' + y = 0, \quad y(0) = 0, \quad y(1) = 1. \quad (3)$$

Its exact solution is given by

$$y(x) = \cos \left( \frac{x}{\sqrt{\epsilon}} \right) + \left[ 1 - \cos \left( \frac{x}{\sqrt{\epsilon}} \right) \right] \frac{\sin(x/\sqrt{\epsilon})}{\sin(1/\sqrt{\epsilon})}. \quad (4)$$

Solution of (3) with $\epsilon = 5 \times 10^{-4}$. 

[Graph of the solution with $\epsilon = 5 \times 10^{-4}$]