Predator-Prey Models

The first predator-prey model we considered was the Volterra model, here presented in dimensionless form:

\[ \dot{N}_1 = N_1 (1 - N_2), \]
\[ \dot{N}_2 = \alpha N_2 (N_1 - 1). \]

A phase plane for this model is graphed below.

![Phase plane for Volterra model, \( \alpha = 1 \).](image-url)
Because of the lack of physical justification for the Volterra model, we tried the following new model:

\[
\dot{N}_1 = N_1 \left( 1 - N_1 - \frac{N_2}{N_1 + d} \right),
\]

\[
\dot{N}_2 = bN_2 \left( 1 - \frac{N_2}{N_1} \right).
\]

We noted that the only nontrivial fixed point changed stability depending on the values of \( b \) and \( d \), as shown in the graph below.

Whenever the fixed point was unstable, we saw that the only possibility (due to inward flow from infinity) was that there must be a stable limit cycle, shown below.