In class, we discussed the solutions of the following system in *action-angle* coordinates:

\[
\dot{r} = f(r), \quad \dot{\theta} = \omega.
\]

We noted that the sign of \(\omega\) determined whether the trajectories spun clockwise or counterclockwise, while the existence and stability of limit cycles could be determined from the \(\dot{r}\) equation.

In all the figures that follow, \(\omega > 0\), so we have counterclockwise rotation.

This figure shows the case where \(f(r) = 1 - r^2\). Here there is a stable limit cycle at \(r = 1\).
This figure shows the case where $f(r) = r(r^2 - 1)$. Here there is an unstable limit cycle at $r = 1$; this is essentially the opposite of the Hopf bifurcation diagram.

This figure shows the case where $f(r) = r(r^2 - 1)^2$. Here there is a semistable limit cycle at $r = 1$. Note the extra windings indicating a very slow approach/departure from $r = 1$. 
The next three diagrams concern the case where
\[ f(r) = -r(r^2 - r + \lambda). \]

\[ \lambda = -1/2. \]

In class we showed that if \( \lambda < 0 \), the origin is an unstable spiral and there is one stable limit cycle. This case is shown above.

\[ \lambda = 0.2. \]

In class we showed that if \( 0 < \lambda < 1/4 \), the origin is a stable spiral, then moving outward one finds one unstable and one stable limit cycle. This case is shown above.
In class we showed that if $\lambda > 1/4$, the origin is stable and there are no limit cycles. This case is shown above.